Expedient Hypersonic Aerothermal Prediction for Aerothermoelastic Analysis Via Field Inversion and Machine Learning

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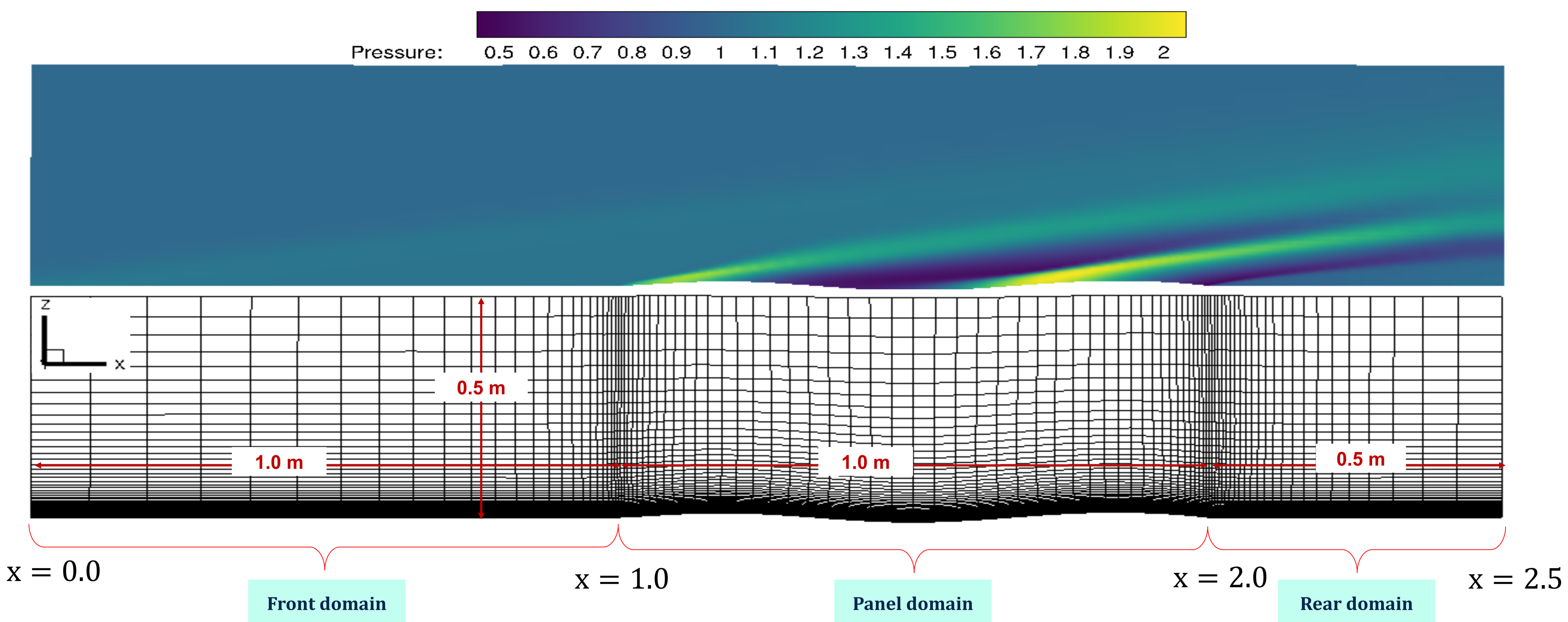
Introduction and Objectives



In the field of engineering, there is always a two-model problem. On one hand, to capture the behavior of a complex physical system, one can develop models of ever-high fidelities, which comes at the cost of high computational cost, sometimes intractable. On the other hand, for fast iterations in engineering design, one can only use models of lower fidelities, which results in products of sub-optimal performance.

This is particularly a problem in the design of hypersonic vehicles, where the interaction between the aerodynamics, structural dynamics, and heat transfer, viz. aerothermoelasticity, becomes important. The goal of our study is to develop a low computational cost but accurate model for aerothermodynamics using a combined strategy of field inversion and machine learning.

Overview of Methodology

Stage 1

High-Fidelity but **Expensive**
RANS Solution

Low-Fidelity but Efficient
Turbulence-Viscous Interaction model

Stage 2

DAE constrained optimization

Stage 3

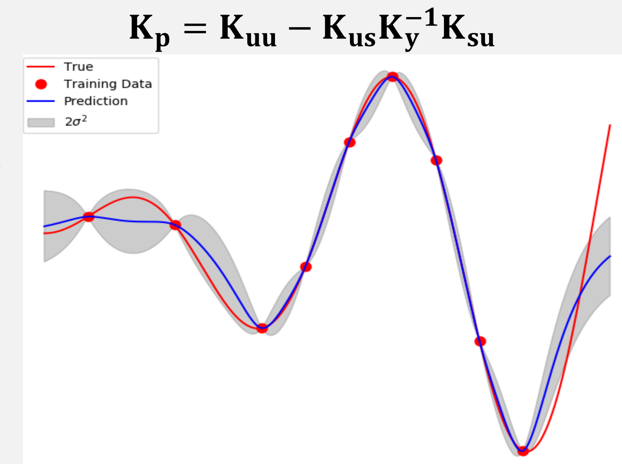
Dataset

y	x	u	θ	β
$y_{1,1}$ ⋮ $y_{n,1}$	x_1 ⋮ x_n	$u_{1,1}$ ⋮ $u_{n,1}$	$\theta_{1,1}$ ⋮ $\theta_{n,1}$	$\beta_{1,1}$ ⋮ $\beta_{n,1}$
$y_{1,2}$ ⋮ $y_{n,2}$	x_1 ⋮ x_n	$u_{1,2}$ ⋮ $u_{n,2}$	$\theta_{1,2}$ ⋮ $\theta_{n,2}$	$\beta_{1,2}$ ⋮ $\beta_{n,2}$
⋮	⋮	⋮	⋮	⋮
$y_{1,n}$ ⋮ $y_{n,n}$	x_1 ⋮ x_n	$u_{1,n}$ ⋮ $u_{n,n}$	$\theta_{1,n}$ ⋮ $\theta_{n,n}$	$\beta_{1,n}$ ⋮ $\beta_{n,n}$

Stage 4

Gaussian Process Regression

Predictive mean:
$$\mathbf{m_p} = \mathbf{m_u} + \mathbf{K_{us}}\mathbf{K_y}^{-1}(\mathbf{y_s} - \mathbf{m_s})$$
Predictive covariance:
$$\mathbf{K_p} = \mathbf{K_{uu}} - \mathbf{K_{us}}\mathbf{K_y}^{-1}\mathbf{K_{su}}$$



Stage 5

High-Fidelity but **Efficient**
Augmented TVI model

The Turbulent Viscous Inviscid Interaction model (TVI) is a simplification of the Navier-Stokes equations that involved multiple assumptions. It consists of three coupled equations relating the primal variables $\boldsymbol{y} = [\delta^*, y_e, P_e]$, which are the boundary layer displacement thickness (δ^*), the boundary layer thickness (y_e), and the boundary layer pressure (P_e). Simulations from RANS and TVI solutions are resolved over the computational domain given in Fig. [1], where a variety of panel deformations were considered.

<u>First-principle modeling</u>: Turbulence-Viscous Interaction (TVI) **Stage 1**

$$\begin{cases} \dfrac{d}{dx}\left(\dfrac{\delta^*}{H}\right) + \left(H_i\dfrac{T_w}{T_o} - 4\right)\dfrac{\delta^*}{HM_e}\dfrac{dM_e}{dx} = \dfrac{C_{fe}}{2} \\ p_e(x) = p_\infty\left(1 + \dfrac{\gamma-1}{2}M_\infty\dfrac{dy_e}{dx}\right)^{\frac{2\gamma}{\gamma-1}} \\ y_e = y_w + \delta^* \end{cases}$$

* Classical integral equations that <u>respect physics</u>
* A system of differential-algebraic equations (DAEs)
* But <u>misses some physics</u>, e.g. High-temperature effects

<u>Model augmentation by Functional correction</u>

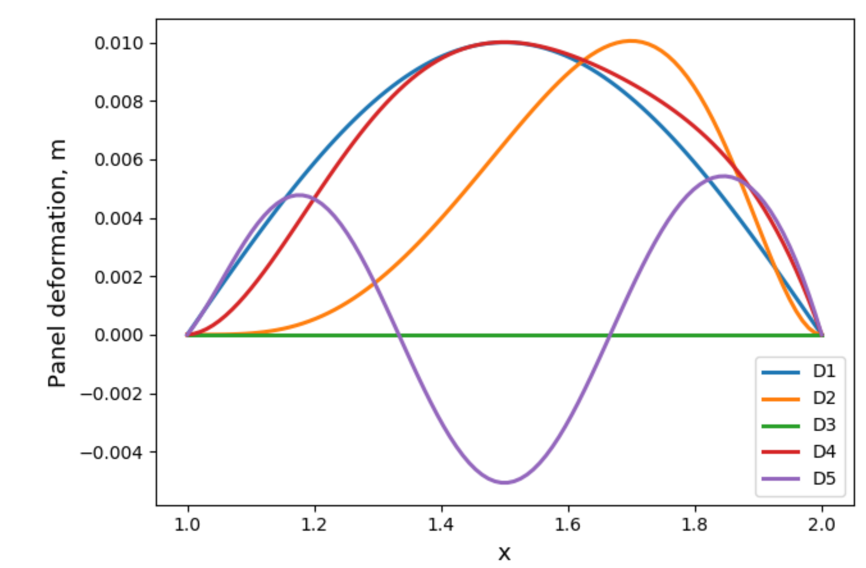
$$\begin{cases} H(x) \leftarrow \beta_H(x)H(x) = \beta_H\left[\dfrac{\gamma-1}{2}M_e^2\left(1 + H_i\dfrac{T_w}{T_o}\right)\right] \\ M_e(x) \leftarrow \beta_M(x)M_e(x) \end{cases}$$

* To account for missing physics.
* Correction for BL shape factor and Mach No. at BL edge
* A new DAE with unknown functions

Stage 2

Inverse Problem and Machine Learning

The goal of the inverse problem is to identify the values of the correctors with which the ATVI solutions can match the RANS primal variables. The correctors are presented using a collocation approach where the values of the correctors $\mathbf{B} = [\beta_{(1)}, \beta_{(2)}, \dots, \beta_M]$ are specified as a set of stations over the solution domain $x_{(1)}, x_{(2)}, \dots, x_{(M)}$. Subsequently, a DAE-constrained optimization problem with the $\mathbf{B}$ parameter as the design variable is formulated for the i[th] operating condition and panel deformation. From where a whole data set can be generated.



Stage 3

$$\mathbf{B}^i = \arg\min_{\mathbf{B}} J\left(\mathbf{y}^i_{\mathrm{RANS}}, \mathbf{y}(\mathbf{B})\right)$$
$$\text{s.t. } \widetilde{\mathbf{M}}(\mathbf{y}; \mathbf{u}, \boldsymbol{\beta})\frac{d\mathbf{y}}{dx} = \widetilde{\boldsymbol{g}}(\mathbf{y}, x; \mathbf{u}, \boldsymbol{\theta}, \boldsymbol{\beta})$$
$$x \in [0, L], \mathbf{y}(0) = \mathbf{y}^i_o$$

Dataset

y	x	u	θ	β
$y_{1,1}$ ⋮ $y_{n,1}$	x_1 ⋮ x_n	$u_{1,1}$ ⋮ $u_{n,1}$	$\theta_{1,1}$ ⋮ $\theta_{n,1}$	$\beta_{1,1}$ ⋮ $\beta_{n,1}$
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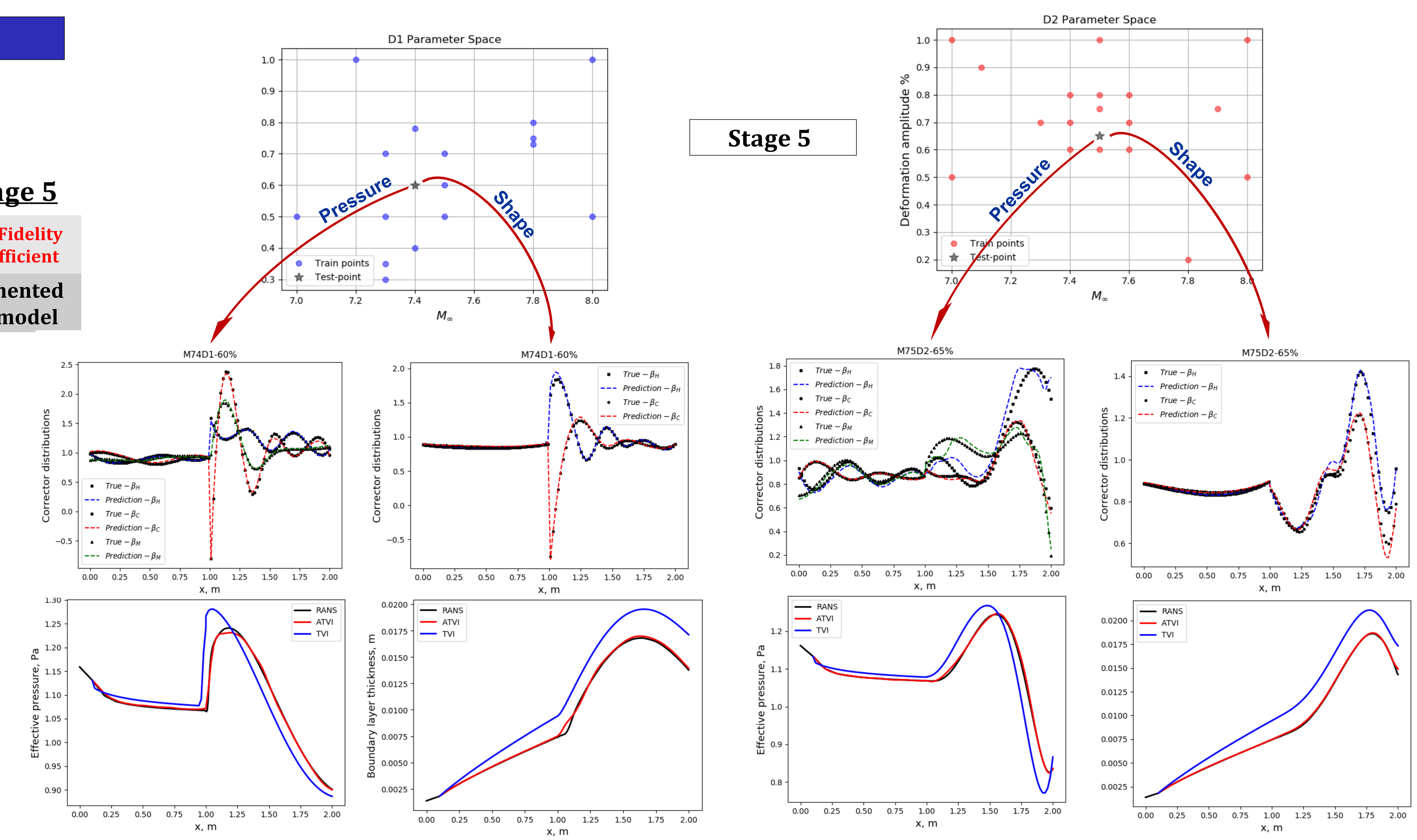
The machine learning method of choice was the Gaussian process regression (GPR) for its robustness in regression and capability to reproduce the error estimate of the prediction. Using the data from Stage 3 as training data, these GPR models were trained to find the correctors β_H, β_C and β_M for a given input. The kernel of choice for the GPRs was the Matern kernel given by,

$$k(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{l}) = \frac{1}{\Gamma(\nu)2^{\nu-1}}\left[\sqrt{2\nu d(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{l})}\right]^\nu \mathbf{K}_\nu\left[\sqrt{2\nu d(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{l})}\right] \quad \text{where,} \quad d(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{l}) = \left[\sum_{i=1}^{n}\left(\frac{x_i - x_i'}{l_i}\right)^2\right]^{\frac{1}{2}}$$

Stage 4

Results

Stage 5



Conclusion

The presented method enhances the predictive capability of a low-fidelity model with the augmentation of correction terms for the missing physics in this model, based on a small amount of high-fidelity solutions. In the aerodynamic application, the ATVI equations significantly outperform predictions from classical TVI equations and can therefore be used as a high-fidelity model to predict boundary layer developments and steady pressure loads over arbitrary structural responses of a hypersonic vehicle.