Expeditious Hypersonic Aerothermodynamics for Aerothermoelastic Analysis Via Field Inversion and Machine Learning

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Overview of Methodology

Stage 1
High-Fidelity but Expensive RANS Solution

Stage 2
DAE constrained optimization

Stage 3
Turbulence-Viscous Interaction model

Stage 4
Gaussian Process Regression
Predictive mean:
\[ m_\beta = m_\beta + K_{\beta} \delta_\beta \]
Predictive covariance:
\[ K_{\beta\delta} = K_{\beta\delta} - K_{\beta\gamma} K_{\gamma\delta}^{-1} K_{\gamma\beta} \]

Stage 5
High-Fidelity but Efficient Augmented TVI model

Results

The goal of the inverse problem is to identify the values of the correctors with which the ATVI solutions can match the RANS primal variables. The correctors are presented using a collocation approach where the values of the correctors \( B = [\beta_1, \beta_2, \ldots, \beta_n] \) are specified as a set of stations over the solution domain \( x_{(1)}, x_{(2)}, \ldots, x_{(M)} \). Subsequently, a DAE-constrained optimization problem with the \( B \) parameter as the design variable is formulated for the \( i \)-th operating condition and panel deformation. From where a whole data set can be generated.

Inverse Problem and Machine Learning

The machine learning method of choice was the Gaussian process regression (GPR) for its robustness in regression and capability to reproduce the error estimate of the prediction. Using the data from Stage 3 as training data, these GPR models were trained to find the correctors \( \beta_\alpha, \beta_\beta, \) and \( \beta_\mu \) for a given input. The kernel choice for the GPRs was the Matern kernel given by:

\[
k(x, x'; I) = \frac{1}{\Gamma(\nu)2^{\nu-1}} \left( \sqrt{2\nu d(x, x'; I)} \right)^\nu K_{\nu} \left( \sqrt{2\nu d(x, x'; I)} \right)
\]

where

\[
d(x, x'; I) = \left[ \sum_{i=1}^{n} (x_i - x_i')^2 \right]^{1/2}
\]

Conclusion

The presented method enhances the predictive capability of a low-fidelity model with the augmentation of correction terms for the missing physics in this model, based on a small amount of high-fidelity solutions. In the aerodynamic application, the ATVI equations significantly outperform predictions from classical TVI equations and can therefore be used as a high-fidelity model to predict boundary layer developments and steady pressure loads over arbitrary structural responses of a hypersonic vehicle.

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Fig. 1

Stage 3

Stage 4

Stage 5

Stage 6